Abstracts for Contributed Talks
Spring 2019 MAA Seaway Section Meeting

1. **Ian Alevy** (University of Rochester)
   
   **Renormalizable Rectangle Exchange Maps**

   A domain exchange map (DEM) is a dynamical system defined on a smooth Jordan domain which is a piecewise translation. We explain how to use cut-and-project sets to construct minimal DEMs. Specializing to the case in which the domain is a square and the cut-and-project set is associated to a Galois lattice, we construct an infinite family of DEMs in which each map is associated to a PV number. We develop a renormalization scheme for these DEMs which we use to study the ergodic properties of these maps. Finally we explain how to compose some of these maps to create multistage, renormalizable DEMs.

2. **Ahmad Almomani** (SUNY Geneseo)
   
   **A Modification Particle Swarm Optimization for Non-Differentiable Test Problems**

   Particle Swarm Optimization (PSO) has introduced by studying the social behavior that implemented rules using distance matching velocity and acceleration of the particles to get swarming behavior in groups of simple agents. But the uncontrolled increase of the velocities values leads to swarm explosion and the sensitivity of the parameters modifications that may profoundly change the convergence of the algorithm.

   In this talk, a new modification for Particle Swarm Optimization (PSO) algorithm is proposed and applied on very well-known non-differentiable benchmark problems where we cannot use any derivative based method. This modification produces high reliability, cost, and robustness of PSO. This approach depends on the update after each iteration for the position and velocity that maximize the diversity between all swarm individuals.

3. **Christopher Beam** (RIT)
   
   **State Estimation for Cardiac Action Potential Dynamics:**

   **A Comparison of Linear and Nonlinear Kalman Filters**

   Cardiac arrhythmias are a leading cause of death in the industrialized world. Various technologies, such as electrocardiography, optical mapping, and patch clamping, have been developed to monitor cardiac electrophysiological behavior in live tissue. One limitation is that none of the available measurement methods is capable of monitoring simultaneously all quantities, such as intracellular ionic concentrations and ion-channel gating states, that may be important contributors to arrhythmia formation. To help fill this gap, we tested two state estimation algorithms on the Karma two-variable model, which is a nonlinear differential equation model of cardiac action potential (AP) dynamics. State estimation algorithms allow for reconstruction of dynamical (or state) variables of a system, based on limited measurements, in cases
where certain state variables cannot be observed directly. While a number of different varieties of Kalman filter (KF) have been produced since the technique's introduction, to our knowledge no cardiac AP studies have been performed comparing the effectiveness of a traditional, linear KF against that of a newer, nonlinear KF. To that end, we estimated the slow variable of a single-cell Karma model from noise-corrupted measurements of the fast variable, and compared estimates from a linear KF with periodic gain updates against those from a nonlinear filter, specifically the Unscented Kalman Filter (UKF). While the linear filter was able to estimate the state of the model well for longer cycle lengths (CLs), in the range of 800 ms, it was outperformed by the UKF for shorter CLs, in the range of 500 ms.

4. **Marlo Brown** (Niagara University)

*Analysis of an Activity in Elementary Statistics*

In the elementary statistics course, I motivate the concepts of probability, the central limit theorem and confidence intervals by having the students roll a pair of dice. But how accurate is this? This talk will explore the mathematics behind the activity.

5. **Jack Graver** (Syracuse University)

*Is the equation dim(S) + dim(S⊥) = dim (V) valid for general inner products over an arbitrary field?*

Let V be a finite dimensional vector space over the field F. By an inner product, we mean a nonsingular, symmetric, bilinear form. Over the reals the symmetric bilinear form is usually required to be positive definite, which makes no sense over most fields. Here positive definite is replaced by the much weaker condition of nonsingularity: the only vector orthogonal to every vector in V is the zero vector. The usual bilinear form, summing the pairwise products of the coordinates, is an inner product in this sense over F, for any field F. The usual proof, over the reals, of this dimension theorem is no longer valid in this general setting. The problem is that there may well be non-zero vectors orthogonal to themselves. When a subspace S contains such vectors, the subspace S∩S⊥ may well have positive dimension and so, V is no longer a direct sum of S and S⊥. For a simple example, consider a finite set X. With Boolean sum (A + B = A ∪ B - A∩B) and the obvious scalar multiplication by scalars in the field $\mathbb{F}_2$, the set of subsets of X is an $|X|$-dimensional vector space over $\mathbb{F}_2$. One easily checks that it admits the inner product defined by $<A,B> = |A∩B|_{\text{mod }2}$. Let $X=\{a,b,c,d\}$ and let S be the 2-dimensional subspace with vectors $\varnothing, \{a,b\}, \{c\}$ and $\{a,b,c\}$. One easily checks that $S⊥$ is the 2-dimensional subspace with vectors $\varnothing, \{a,b\}, \{d\}$ and $\{a,b,d\}$. It follows that $S∩S⊥$ is the 1-dimensional subspace consisting of the vectors $\varnothing$ and $\{a,b\}$. At least in this example, dim(S) + dim(S⊥) = dim (V) still holds. But, is it always valid and, if it is, can we prove it?

6. **Marvin Gruber** (RIT)

*Inverse Problems, Tikhonov Regularization and Ridge Regression*

Let X and Y be two spaces where the elements of X are unknown and the elements of Y are known or available. Let T be a mapping from X to Y. Solving an inverse problem consists of finding elements \(x\) in X such that \(Tx = y\) where y is an element of Y. Two examples of inverse problems are:
1. When $X$ and $Y$ are Hilbert spaces and $T$ is a linear transformation between them;
2. Fitting a linear regression model given a data set.

The inverse problem in 1 above is well posed if:
1. there exists one solution $x$;
2. the solution $x$ is the only solution;
3. the solution $x$ depends continuously on $y$.

When one or more of the three conditions above does not hold the problem is ill-posed. For an ill-posed problem a small change in $y$ might mean a large change in $x$.

Consider a linear model of the form $Z = W\beta + \varepsilon$ where $Z$ is a vector of observations of a variable that depends on a matrix of fixed values $W$ or observations of one or more variables. The problem here is finding a vector of $\beta$ coefficients that gives an appropriate fit of the model.

For data where there are high correlations between some the $W$ variables (multicollinear data) the least square estimator can have very high variability. One way to solve the first problem is Tikhonov regularization. A possible solution to the second problem is to use ridge regression estimators. Ridge regression is Tikhonov regularization for the finite dimensional case. As might be expected many of the ideas and concepts that pertain to these two problems and their solutions are very similar. The goal of the talk is to compare and contrast them.

7. Aaron Heap (SUNY Geneseo)

*Space-Efficient Knot Mosaics*

Knot mosaics, which are representations of knot diagrams on a square array created using tiles chosen from a specific list of eleven mosaic tiles, were first introduced by Lomonaco and Kauffman in 2008. They introduced the mosaic number of a knot, which is the smallest size mosaic on which the knot can be represented. We introduce the concept of a space-efficient knot mosaic, which uses the least number of non-blank tiles necessary to depict the knot. This least number is called the tile number of the knot. In this talk, we will discuss these introductory concepts, provide strict bounds for the tile number of a knot in terms of the mosaic number of the knot, and give a complete list of prime knots with mosaic number six or less. Knot mosaic theory is a great source of research projects that are accessible to undergraduates, and we will discuss a few of these unanswered questions.

8. Yozo Mikata (Fluor Corporation)

*1D Phononic Metamaterials: Shear Waves*

Metamaterials have been studied extensively since Pendry and Holden (1999), and Smith et al. (2000) have succeeded in creating an electromagnetic composite material (photonic metamaterial) with negative permeability and negative permittivity in certain frequency range (double negative materials). But the
original concept goes back to a theoretical paper by Veselago in 1967. In this talk, binary and ternary 1D phononic metamaterials will be discussed in relation to local resonance. Particular attention will be focused on the effect of the geometrical parameters of the material periodicity on the dispersion characteristics of shear (SH & SV) waves. Both infinitely layered material and finitely layered material will be considered. For infinitely periodic materials, dispersion relations will be obtained, and for finitely periodic materials, transmission coefficients will be discussed. The effect of the incidence angle will be also discussed.

9. **Sedar Ngoma** (SUNY Geneseo)

*On a space-dependent inverse source problem for a parabolic Equation*

We consider an inverse source problem for a parabolic partial differential equation in which the source function depends only on the space variables in a domain $\Omega$ of $\mathbb{R}^d$, $d \geq 1$. To recover this source function, the equation is subject to an integral constraint and the final time overdetermination. We show the existence, uniqueness, and stability of classical solutions in Hölder spaces. Our numerical scheme uses a finite element discretization in space to approximate the solution of this inverse problem. The numerical results and the error reported show the accuracy of our approximation.

10. **Robert Rogers** (SUNY Fredonia)

*Stop Teaching the Chain Rule! (And Still Teach Calculus)*

Did the title catch your attention? How many of your calculus students struggle with the chain rule? This talk will demonstrate that by focusing on differentials rather than derivatives, the chain "rule" can be replaced by the broader and more natural technique of substitution. We will also see how topics such as related rates and implicit differentiation become more natural when focusing on differentials. We will look at how this affects the multivariable calculus chain rule as well.

11. **Simon Romero** (Alfred University)

*Using Board Games in an Inquiry-Based Learning Graph Theory Class*

When teaching Mathematics, it is often overlooked how barren the presentation of concepts are. The “User-Interface” when teaching Mathematics could be one of the reasons for the lack of interest of some of our students. In particular, one of the problems of teaching a Graph Theory (to non-Mathematics majors) is the amount of terminology, concepts and methodology that students need to assimilate in a short period of time. The presenter has taught a Graph Theory class where board games are used to ease the barrier of entry previously described. The games used, class structure and the experiences will be presented.
12. **Philippe Savoye** (Mansfield University)

*Introducing Students to Discrete-Time Problems in Differential Equations Classes.*

Many benefits can be obtained by illustrating how commonly taught methods in introductory ordinary differential equations courses can be extended to discrete-time problems. The method of undetermined coefficients, for example, can be adapted to solve non homogeneous difference equations. Similarly, the use of the z-transform as a tool in solving discrete-time initial value problems can be developed in parallel with the Laplace transform. Incorporating these topics prepares students to examine issues arising in modern signal processing applications.

13. **Robert Sulman** (SUNY Oneonta)

*Linear Functions (modulo n)*

We consider linear functions \( f(x) = ax + b \) with coefficients in \( \mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\} \), the (least non-negative) complete residue system (mod \( n \)). This setting induces a collection of “cycles” resulting from iteration of \( f \) starting with some element \( x_0 \) of \( \mathbb{Z}_n \). A “\( k \)-cycle” is a set of distinct elements of \( \mathbb{Z}_n \): \( x_0, f(x_0), \ldots, f^{k-1}(x_0) \), where \( f^{k}(x_0) = x_0 \). The fixed points (one-cycles) of \( f \) play a prominent role in what follows. We explore the cycle structures of such functions \( f \) when \( a \in (\mathbb{Z}_n)^* \), the group of units of the ring \( (\mathbb{Z}_n, +_n, \cdot_n) \) as well as when \( \gcd(a, n) > 1 \). In the former case, a complete cycle structure can be determined when both \( a \) and \( a-1 \) lie in \( (\mathbb{Z}_n)^* \) and more generally, when \( \gcd(a-1, n) \) divides \( b \). When \( \gcd(a, n) > 1, f \) is not injective and its cycle structure contains “whiskers” which are distributed evenly among the cycles. We also examine the group “\( \text{Lin}(\text{mod } n) \)” of such linear functions with \( a \in (\mathbb{Z}_n)^* \). In this segment of the talk we see that this group is a subgroup of the Symmetric Group, \( S_n \). In fact, \( \text{Lin}(\text{mod } n) \) is a semi-direct product. We explore and prove several results about this group involving its Center Conjugacy, and look at Centralizers of specific \( f \in \text{Lin}(\text{mod } n) \) for specific \( n \). The very description of \( f \) facilitates its analysis.

This talk is accessible to undergraduate students who have some Elementary Number Theory background and possibly some basic Group Theory background.